Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name:Group Theory

Subject Code: 4SC05GTC1 **Branch: B.Sc.(Mathematics)**

Semester: 5 **Date: 24/03/2017** **Time:02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Prove that if a, b, c are in G, then $ab = ac \Rightarrow b = c$. **(02)**

b) Examine whether the following permutation is even or odd : **(02)**
$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 5 & 4 & 3 & 6 & 1 & 7 & 9 & 8 \end{pmatrix}$.

c) Show that every subgroup of an abelian group is normal. **(02)**

d) Define: Order of an element.If$G = \{1, -1, i, -i\}$ is a multiplicative group, then find order of $-i$. **(02)**

e) If H and K are finite subgroups of a group G. If$o(H)$&$o(K)$ are respectively prime integers then show that $H \cap K = \{e\}$. **(02)**

f) Define: Normal subgroup. **(01)**

g) Define: Index of Hin G. **(01)**

h) The intersection of two subgroups of a group is also a subgroup. Determine whether the statement is True or False. **(01)**

i) Every cyclic group is an abelian group. Determine whether the statement is True or False. **(01)**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Show that the set of positive rational numbers becomes a commutative group under binary operation $'*'$ where $a * b = \frac{ab}{2}$. **(05)**

b) Prove that in a group G, the equations $a * x = b$ and $y * a = b$, where $a, b \in G$ have unique solution. **(05)**

c) Prove that if the element a of a group G is of order n, then $a^m = e$ if and only if n is a divisor of m. **(04)**

Q-3 **Attempt all questions** **(14)**

a) Show that $G = \left\{\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \middle| \ a \ is \ any \ non-zero \ real \ number\right\}$ is a commutative **(05)**

group under matrix multiplication.

b) Define: Cyclic group. Show that the group $(\{1,2,3,4,5,6\},\times_7)$ is cyclic. How many generators are there? **(05)**

c) Define a permutation. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$, find AB and BA. **(04)**

Q-4 **Attempt all questions** **(14)**

a) If $G = \langle a \rangle$ is a cyclic group of order 12, then obtain all subgroups of G and prepare their lattice diagram. **(05)**

b) Show that $a \to a^{-1}$ is an automorphism of a group G if and only if G is abelian. **(05)**

c) Prove that if H, K are two subgroups of a group G, then HK is a subgroup of G if and only if $HK = KH$. **(04)**

Q-5 **Attempt all questions** **(14)**

a) If R is the additive group of real numbers and R_+ the multiplicative group of positive real numbers, then prove that the mapping $f: R \to R_+$ defined by $f(x) = e^x$, for all $x \in R$ is an isomorphism. **(05)**

b) Prove that a subgroup H of a group G is a normal subgroup of G if and only if each left coset H in G is a right coset of Hin G. **(05)**

c) Define the binary operation $*$ on $\mathbb{Z}$ as $a * b = \max\{a, b\}$. Examine the binary operation $*$ for commutativity, associativity and identity element. **(04)**

Q-6 **Attempt all questions** **(14)**

a) Show that the set $G = \{[0],[1],[2],[3],[4],[5]\}$ is a finite abelian group of order 6 with respect to addition modulo 6. **(05)**

b) Prove that every subgroup of a cyclic group is cyclic. **(05)**

c) Using the Euler's theorem, find the remainder obtained on dividing 3^{256} by 14. **(04)**

Q-7 **Attempt all questions** **(14)**

a) State and prove Lagrange's theorem. **(07)**

b) State and prove Cayley's theorem. **(07)**

Q-8 **Attempt all questions** **(14)**

a) Prove that the order of a permutation $f \in S_n$ is the least common multiple of the lengths of its disjoint cycles. **(07)**

b) State and prove first fundamental theorem of Homomorphism. **(07)**